**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

**Sol.** Commitment time is 1 hour=60 minutes.

He start working on after 10 minutes car dropped off. Then, remaining value time is = 50 minutes.

Probability that the service manager meet his commitment = 1-pnorm(50,45,8)=0.27 (approx)

1. 0.3875
2. 0.2676
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44. FALSE
7. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees. TRUE.

**Sol.** Z=30-38/6=-1.33

P(-1.33)=0.092

Expected value is = 0.092\*400=36.8

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Sol.** As per the information X1 and X2 are mutually independent and identically distributed random variables. Hence, average and variance will be same for both. As these are mutually independent previous output does not affect current output and as these are identically distributed chances getting particular outcome is same.

For example, X1 = c(4,10,12), Avg=8.6

X2 = c(12,10,4) , Avg=8.6

2X1=c(8,20,24),Avg=17.33, has high variance

X1+X2=c(16,20,16),Avg=17.33 has less variance

Both X1 and X2 following normal distribution while variance parameter is different.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Sol.** Avg=100,SD=20, 99% confidence interval = 2.576

Confidence Interval formula = Avg +/- Z(SD/SQRT(N))

100+2.576(20)=151.52

100-2.576(20)=48.48

Confidence interval range =[48.5,151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Sol.**

Profit1 Avg = 5 million Dollors=5\*45=225 million rupees

Profit2 Avg = 7 million Dollors=7\*45=315 million rupees

Total company profit = (225+315)=540 million rupees

Profit variance (SD) = 45\*sqrt(9+16)=45\*5=225 million rupees

1. Annual company profit in rupees range at 95% confidence interval is :

[540-(1.96\*225),540+(1.96\*225)]=[99, 981] (in millions)

1. 5th Percentile = 90% Probability

Annual company profit in rupees range at 90% confidence interval is :

[540-(1.645\*225),540+(1.645\*225)]=[169.875, 910.125] (in millions)

1. Profit 1 Z-score=(0-5)/3=-1.667

Profit 2 Z-score=(0-7)/4=-1.75

**Division 2** has larger probability of making a loss in a given year